CORPORATE OPERATING INCOME FORECASTING ABILITY

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Recently, interest has materialized in the business, investment and governmental communities regarding the propriety, possibility and potential consequence of allowing or requiring publicly held corporations to routinely report income forecasts. The SEC, for example, has conducted rather extensive formal hearings to assess the attitudes, arguments and problems that might arise if accounting-based income projections or other forecasts such as expected capital expenditures are routinely included in annual financial statements (10-Ks) filed with the Commission. The SEC has tentatively concluded that corporations falling under its jurisdiction will be permitted, but presently not be required, to incorporate income projections [11].

Empirical study presupposes that public reports be decomposed into a number of properties and attributes. At least three important underlying income forecasting properties can be identified: feasibility, reliability and validity.

Feasibility has two interrelated attributes: ability and capability. Forecasting ability specifies the forecasting error (accuracy and variability) that results from applying forecasting models or methods (mechanical and judgmental) to a set of data to create income projections. Forecasting ability can be established by ranking various models in terms of their forecasting error and variability. Elton and Gruber [2] used the ability criterion solely to decide whether security analysts produce more accurate income projections than those generated by mechanical methods. They concluded that there was no significant difference in terms of the accuracy of forecasting error between income projections of analysts and mechanical methods. Both ability and capability considerations must be considered. Capability, as used here, focuses on the annual incremental cost of deploying resources, such as additional man hours and so forth, to prepare, publish and revise annual income forecasts for investors and others. The significance of incremental capability costs seems to be one of the major concerns underlying the SEC's decision to permit, but not require, corporations to include income projections in required filings [10]. If the estimator is corporate management, then incremental capability costs that must be incurred to produce income budgets solely for internal use would be excluded from the analysis. Once accumulated the annualized incremental capability costs directly associated with income forecasts should be related to the measures of forecasting ability. The determination of feasibility is a form of cost/benefit analysis where cost references incremental forecasting capability, and benefit denotes the forecasting error and variability that results from a given forecasting model.

An analysis of ability and capability provides a basis for ranking various income forecasting models. In this context the feasibility criterion constitutes a minimum hurdle. Although providing a sufficient reason, forecasting feasibility is not a necessary basis for deciding whether or not income forecasts ought to be published. Reliability and validity considerations must resolve this issue.

Reliability has two attributes. The variability of a numerical outcome that results from repeatedly applying the same measurement or set of measurement rules is called measurement bias. A second source of unreliability has been discussed traditionally in accounting under the "consistency" rubric. Income statements or projections for adjacent periods are inconsistent if (1) different measurement principles are used or (2) magnitudes of different phenomena are aggregated in different ways. Although the inherent instability of various mechanical forecasting models may not be unduly troublesome, the inconsistency of income estimates generated by judgmental methods may be quite significant, if for no other reason than that there are few, if any, generally agreed measurement rules. Reviewing real estate and actuarial science literature may provide clues about how to construct consistency criteria for judgmental forecasting methods. The creation of such criteria may prove helpful in deciding the kind and degree of disclosure of assumptions underlying income forecasts that must be published.

A third property of income forecasts is their predictive and construct validity. The correspondence between a guess about the value of one variable, such as the future price of a share of common stock, from the expected magnitude of future annual income is termed predictive validity. After establishing the predictive validity of a forecasting model, construct validity should be investigated. Construct validity studies of income forecasting may be used to determine the degree to which a particular predictive method does only what it is designed to do, as well as its usefulness in elaborating, say, accounting theory, micro-economic theory and investment portfolio management theory. Only at this advanced stage can empirically grounded inferences concerning the correctness of permitting or requiring corporations to routinely publish income forecasts be placed in an appropriate theoretical context.

The empirically based research strategy outlined above can be explored in a specified sequential manner. This approach states that the feasibility of permitting or requiring corporate managers to routinely report income projections should be established before more abstract reliability and validity considerations are explored. This research strategy attempts to implement the theory of informational value in a particular setting [4]. Henceforth this paper discusses one aspect of forecasting feasibility, mechanical forecasting ability. A subsequent paper will report the results of an empirical study of corporate personnel which seeks to more fully understand judgmental forecasting ability and various aspects of income forecasting capability.

Mechanical Forecasting Ability

Net operating income was chosen for use in this study rather than net income because it is generally considered to be more stable than final net income, and because it is thought to constitute a better basis for calculating a company's normalized long-run profit potential or earning power [6, chapter 9; 7, pp. 324-25].

In recent years a few studies have attempted to forecast the earning potential or earning power of corporations by employing mechanical forecasting models [1, 2, 5, 9]. Usually, a mechanical forecasting model has been fitted to a time series of income data. A similar approach is employed in this study. The rationale for testing various mechanical forecasting models includes: (1) developing benchmarks against which the forecasts of management can be compared and (2) gaining insight into the nature of the stochastic processes that combine to produce net operating income.

Three different mechanical projection approaches are utilized to forecast a time series of net operating income, NOI, and two models are used to project several NOI components. The components forecasted are sales revenue, cost of goods sold net of depreciation, depreciation and operating expense.

Companies Included in the Study

The data employed in this study were obtained from Standard and Poor's Compustat Tape for the years 1951-71. The study is limited to 68 companies in 10 industries for which at least 19 years of data could be obtained. The 19 year requirement insured that sufficient data existed to estimate parameters and the ability measures with confidence. The industries studied are packaged foods, dairy products, canned foods, drugs, machine tools, specialty machines, office and business equipment, auto parts and accessories, retail department stores and retail food stores.

The industries studied were not randomly selected. A Compustat listing of companies by industry was used to select those thought to possess either a great deal or very little NOI stability. The four industries thought to have unstable NOI flows during the forecast period were machine tools, specialty machines, office and business equipment as well as auto parts and accessories. Because judgmental procedures were employed to select the companies, care must be taken in attempting to generalize the results of this study.

The Forecasting Models

A recursive discounted least-squares forecasting model was used to estimate each NOI component and NOI. This model discounts the data as it ages and revises the parameter estimates with each observation. For example, this means that the forecast for year 19 utilizes the data from years 1 through 18 and that the forecast for year 18 incorporates data from years 1 through 17. In the studies conducted by Elton and Gruber [2] and Frank [5], a similar model provided the best estimate of the accounting data examined. The two versions of the model utilized are formally defined in the Appendix.

The data for each company were separated into two groups. The first 10 years of data were utilized to fit a constant and linear model to each NOI component. The standard error of the estimate was used to choose between the two versions of the recursive discounted least-squares model to forecast the NOI components for the remaining 10 years.

Three approaches were used to forecast NOI flows. First, NOI was directly forecasted as a time series. Second, it was forecasted indirectly as a function of sales revenue. With this approach a forecast of NOI is developed by first forecasting a value for sales revenue and subsequently forecasting NOI as a function of revenue. A third approach viewed NOI as a residual. With this approach NOI is forecast by first forecasting the NOI components, then NOI is calculated as sales revenue less the sum of the cost of goods sold including depreciation and the operating expense.

Measures of Mechanical Forecasting Ability

The measure of mechanical forecasting accuracy used in this study is the mean absolute percentage difference between the actual and forecasted number for a time series. By using a percentage of the actual net income, differences between firms and different time series can be compared since the magnitudes of the numbers are normalized. The average absolute percentage error is computed by using data from years 11 through 20 since years 1 through 10 are employed to estimate the coefficients in the forecasting models.

While the average accuracy of a forecasted time series is an important attribute of mechanical forecasting ability, the concept of variability cannot be ignored because averages can be deceptive. For example, an average annual 15% absolute forecasting error may consist of a series of 5 and 25% errors or a series of 15% errors. For this reason the variability of the forecasting errors should be examined. It is assumed that the less the variability the more useful will be the forecasted data. To measure the variability of NOI forecasts a statistic similar to the standard error of the estimate was used. It is a variant of Theil's U statistic. It expresses the squared forecast error as a fraction of the squared actual annual earnings. A similar version of the U statistic was used in the Elton and Gruber study [2].

Results

The results of applying two versions of the recursive discounted least-squares forecasting model to the estimation of the NOI components and NOI are presented in Exhibits 1-10. Exhibit 1 contains the mean absolute percentage error, MAPE, by industry for each NOI component and three different approaches to estimating NOI. Exhibits 2-8 contain the cumulative MAPE by industry. Exhibits 9 and 10 contain the U statistic which measures the variability of the forecasts of the NOI components and NOI. These exhibits are summarized below.

Accuracy

Exhibit 1 summarizes the MAPE by industry on four major NOI components and for three different approaches to forecasting NOI. An inspection of this exhibit's column totals reveals that the forecasts of the NOI components are more accurate than the NOI forecasts. This expected result occurs because the components are combined to produce NOI, and because NOI typically is not as large as many of the components. Thus, as NOI approaches zero the value of the MAPE increases without a proportional change in the size of its deviation.

Three of the six industries thought to be stable—dairy products, drugs and packaged foods—produced a MAPE of 10% or less for both the NOI and the NOI components. The other three industries thought to possess stable income flows—canned foods, retail department stores and retail food chains—had a MAPE of 24.0, 10.7, and 16.7%, respectively, for NOI as a time series, and a MAPE of less than 10% for the NOI components. The four volatile industries selected—auto parts, machine tools, office and business equipment and specialty machines—produced a MAPE of greater than 20% for each of three approaches to forecasting NOI and a MAPE greater than the average of the ten industries studied for the NOI component forecasts. Among the volatile industries, the NOI component MAPEs are much smaller than the MAPEs for the NOI forecasts. This suggests that the NOI components can be represented better as a time series than NOI for the volatile industries for the time period investigated.

Exhibits 2-8 contain the cumulative MAPE for NOI components. In each of these exhibits the industries are rank ordered in terms of the ability of the mechanical models to forecast NOI and the NOI components within a 10% or less and MAPE (See the 10% column in Exhibits 2-8). The assumed volatile industries ranked last for sales, cost of goods sold net of depreciation and NOI as a time series. For depreciation and operating expense the volatile industries ranked in the last five places for

Exhibit 1. Mean Absolute Percentage Error

Industry	Sales	Cost of goods sold net of depreciation	Depreciation	Operating expenses	Net operating income as a time series	Net operating income as a per cent of sales	Net operating income as a residual
Stable							
Canned foods	.046	.051	.099	.044	.240	.244	.250
Dairy products	.034	.032	.048	.040	.090	.096	.096
Drugs	.058	.101	.082	.059	.076	.076	.082
Packaged foods	.051	.060	.077	.053	.076	.079	.086
Retail dept. stores	.043	.045	.067	.095	.107	.117	.132
Retail food chains	.041	.041	.047	.050	.167	.171	.201
Volatile							
Auto parts	.101	.093	.096	.124	.464	.477	.476
Machine tools	.157	.140	.135	.135	.368	.377	.383
Office and business equip.	.086	.106	.104	.108	.232	.242	.234
Specialty machines	.095	.101	.102	.076	.230	.245	.240
Total — all industries	.073	.080	.087	.078	.212	.219	.223

Exhibit 2. Sales

Industry[1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Canned foods	72*	100*	100*	100*	100
2. Dairy products	100*	100*	100*	100*	100
3. Packaged foods	43	100*	100*	100*	100
4. Retail food chains	58*	100*	100*	100*	100
5. Drugs	40	90*	100*	100*	100
6. Retail dept. stores	75*	75	100*	100*	100
7. Specialty machines	13	63	88	100*	100
8. Office & business equip.	20	60	80	100*	100
9. Auto parts	0	56	78	100*	100
10. Machine tools	0	33	33	83	100
Total—all industries	38	78	88	98	100
Corporate sales—FEI study	53	84	93	95	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.
*The mechanical models produced more accurate forecasts than those reported in the FEI study.

Exhibit 3. Cost of Goods Sold Net of Depreciation

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Dairy products	100	100	100	100	100
2. Packaged foods	44	100	100	100	100
3. Retail food chains	58	100	100	100	100
4. Canned foods	57	85	100	100	100
5. Retail dept. stores	75	75	100	100	100
6. Drugs	10	60	80	80	100
7. Office & business equip.	0	60	60	100	100
8. Auto parts	0	44	88	100	100
9. Specialty machines	13	38	75	100	100
10. Machine tools	0	33	50	83	100
Total—all industries	28	68	86	96	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

Exhibit 4. Depreciation

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Dairy products	60	100	100	100	100
2. Retail food chains	58	100	100	100	100
3. Retail dept. stores	25	75	75	100	100
4. Drugs	10	60	100	100	100
5. Packaged foods	43	57	86	100	100
6. Auto parts	12	56	78	89	100
7. Canned foods	28	43	86	86	100
8. Office & business equip.	0	40	80	100	100
9. Specialty machines	13	25	100	100	100
10. Machine tools	0	17	50	100	100
Total—all industries	24	56	87	97	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

Exhibit 5. Operating Expense

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Canned foods	43	100	100	100	100
2. Dairy products	60	100	100	100	100
3. Retail food chains	72	100	100	100	100
4. Drugs	50	90	100	100	100
5. Packaged foods	43	86	100	100	100
6. Specialty machines	25	62	100	100	100
7. Machine tools	0	50	50	83	100
8. Retail dept. stores	50	50	75	100	100
9. Auto parts	22	44	77	89	100
10. Office & business equip.	0	40	80	100	100
Total—all industries	37	74	90	97	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

Exhibit 6. Net Operating Income as a Time Series

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Packaged foods	29	86	86	86	100
2. Dairy products	20	60	80	100	100
3. Drugs	30	60	100	100	100
4. Retail dept. stores	0	50	75	100	100
5. Canned foods	0	14	57	71	100
6. Retail food chains	14	14	86	86	100
7. Specialty machines	13	13	38	63	100
8. Auto parts	0	0	0	11	100
9. Machine tools	0	0	17	17	100
10. Office & business equip.	0	0	40	60	100
Total—all industries	12	30	58	70	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

Exhibit 7. Net Operating Income as a Per Cent of Sales

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Packaged foods	29	86	86	100	100
2. Drugs	30	60	100	100	100
3. Retail dept. stores	0	50	75	100	100
4. Dairy products	20	40	80	100	100
5. Specialty machines	13	25	25	50	100
6. Canned foods	0	14	57	71	100
7. Retail food chains	14	14	72	86	100
8. Auto parts	0	0	0	11	100
9. Machine tools	0	0	17	17	100
10. Office & business equip.	0	0	40	60	100
Total—all industries	12	30	55	68	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

Exhibit 8. Net Operating Income as a Residual

Industry [1]	Cumulative mean absolute percentage error				
	5%	10%	15%	20%	21+%
1. Dairy products	20	60	80	100	100
2. Drugs	30	60	90	100	100
3. Packaged foods	43	57	86	100	100
4. Retail dept. stores	0	50	50	75	100
5. Specialty machines	13	25	25	50	100
6. Canned foods	0	14	57	71	100
7. Retail food chains	14	14	57	86	100
8. Auto parts	0	0	0	11	100
9. Machine tools	0	0	17	17	100
10. Office & business equip.	0	0	40	60	100
Total—all industries	13	28	50	69	100

[1] The industries are rank ordered in terms of the 10% cumulative error column.

both NOI components along with canned foods (depreciation) and retail department stores (operating expense), respectively. For NOI as a percent of sales and as a residual, one volatile industry, specialty machines, ranked fifth for both methods of forecasting NOI.

Exhibit 2 along with the cumulative MAPE also contains the forecasted cumulative MAPE for sales reported in a recent FEI study [8, pp. 44, 52, 53]. [Care must be taken interpreting this study since the FEI opposes the public reporting of forecasted information.] Comparing the cumulative MAPE for sales with the results of the FEI study, Exhibit 2, reveals that the mechanical models appear to produce a forecast which is at least as accurate as company forecasts in selected industries. At the 5% cumulative MAPE level, four industries produced mechanical forecasts that were better than the FEI average, and at the 15% cumulative MAPE level, six industries produced mechanical forecasts which were better than the reported FEI averages. These industries are identified in Exhibit 2.

Examining Exhibits 2-5 reveals that the forecasts of sales, cost of goods sold (net of depreciation), depreciation, and operating expense were within at least 15% of the actual observations for over 85% of the companies. At the 10% cumulative MAPE level, sales included 78% of the companies, cost of goods sold net of depreciation 68%, depreciation 56%, and operating expense 74%.

The results of three approaches to forecasting NOI, described in Exhibits 6-8, were not as accurate as the forecasts of the NOI components. The forecasts of NOI as a time series produced the most accurate results. Approximately 58% of the time, series forecasts were within 15% of the actual observations, while only 30% were within 10% of the actual observations.

Variability

The U statistic was computed to measure the variability of the NOI components and NOI. The range of the U statistic for the NOI components is shown in Exhibit 9. There is relatively little variability in the forecasts of NOI components. The range for the cost of goods sold and operating expense is greater than that of the other two components. This difference is explained because two companies generated a large U value relative to the values of U generated by the other companies. By eliminating these two extreme values, the U ranges for the cost of goods sold and operating expense are similar to the other two components. This is illustrated by the bracketed numbers in Exhibit 9.

The U statistic is summarized in Exhibit 10 for the three approaches to forecasting NOI. Generally, the U statistic is higher for the three approaches of forecasting NOI than for the NOI components. This was expected because the components that combine to generate NOI are highly correlated and the variance of the sum or differences for correlated data is greater than the variance of uncorrelated data [3,p.230]. The high values in Exhibit 10 are the result of one company. Elimination of this company narrows the range of the U statistic considerably. The numbers in brackets demonstrate the result of eliminating this company.

The analysis of the accuracy and variability of the net operating income data for the companies studied indicates that mechanical forecasting models produced in this study forecasts with a fairly small range of error for the NOI components. However, the mechanical approaches employed produced a wider range of error for several types of NOI forecasts relative to the NOI component forecasts.

Exhibit 9. U Statistic Net Operating Income Components

Component	Low	High	Range
Sales	.00037	.08174	.08137
Cost of goods sold	.00014	.22517 (.09772)	.22503 (.09758)
Depreciation	.00031	.09849	.09818
Operating expense	.00048	.17134 (.08164)	.17086 (.08116)

Exhibit 10. U Statistic Operating Income

Operating income	Low	High	Range
Time series	.00074	1.86734 (.58110)	1.86660 (.58036)
Per cent of sales	.00083	.92747 (.64612)	.92664 (.64529)
Residual	.00091	1.90563 (.60206)	1.90472 (.60115)

Some Observations

Given sampling limitati ·ns, our conclusions about forecasting ability can be but suggestive at this juncture. In terms of forecasting ability, the results of this initial undertaking suggest that there may be significant industry differences in mechanically projecting NOI and several important components of NOI. If significant differences exist in the ability of industries to mechanically forecast NOI and its components, these differences might serve as one guideline to assist in deciding the type of forecasted information a company should be permitted or required to publicly report. For instance if a company has a relatively stable and extended earnings history which can be forecasted with mechanical models, then there may be no necessary reason to permit such a company to publicly report judgmental accounting income estimates, since the informational value provided to the investment community is likely to be slight. In other industries where mechanical models do not produce as good a forecast, factors not included in the forecast model apparently are affecting the financial data. In these instances, perhaps management should be permitted to report the forecasted financial impact of these factors along with detailed assumptions. Also if mechanical models are being utilized and corporate officials have knowledge of significant expected events, such as strikes, legislation, and so forth, that could alter the historical financial trends, then judgmental forecasts, along with the necessary assumptions, might be necessary to indicate, for example, a turning point in the time series. The type of data presented in Exhibit 1 might prove to be of some use in developing disclosure guidelines for companies and/or industries in relation to the public presentation of forecasted financial data. This suggestion differs from that offered by Daily [1]. He implies that a single criterion should be applied to all industries.

The results suggest that it may be possible to forecast the NOI components more accurately than NOI. This could be due in part to the fact that NOI is a function of serveral variables, all of which are highly correlated. Also, since NOI is a residual of its components it tends to be a smaller number, and as this number approaches zero the percentage error will increase without a proportional change in the size of the error. This tends to affect NOI because many companies have components of their cost structure, due to operating and financial leverage, that are not responsive to changes in volume.

Finally, the oft-heard argument that requiring corporations to routinely publish forecasted annual income figures or income components will place such companies at a competitive disadvantage would not seem to be valid for industries where mechanical models perform at acceptable levels of forecasting accuracy and variability, if for no other reason than the mechanical forecasting models employed in this study are well documented in the literature.

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Appendix. Forecasting Models

Each component of operating income is represented by the time series model described in equation (1).

$$x(t) = a_1 f_1(t) + a_2 f_2(t) + \dots + a_n f_n(t) + E(t) \tag{1}$$

where:

$x(t)$ = value of an operating income component at time t,
a_i = coefficients,
$f_i(t)$ = any function of time,
$E(t)$ = random component.

The time function, $f_i(t)$, used in this study includes a constant and a linear model. Equation (2) represents the constant model; and equation (3) represents the linear model.

$$x(t) = a(t)(1) + E(t) \tag{2}$$

$$x(t) = [a_1(t), a_2(t)]\begin{bmatrix} 1 \\ t \end{bmatrix} + E(t) \tag{3}$$

To develop forecasts of the components of operating income, equation (4) is used.

$$x(t+\gamma) = \underline{\hat{a}}(t)\,\underline{f}(t+\tilde{\iota}) \tag{4}$$

where:

$\hat{x}(t+\tilde{\iota})$ = forecast for $\tilde{\iota}$ periods ahead,
$\underline{\hat{a}}(t)$ = estimate of the value of the coefficient,
$f(t+\tilde{\iota})$ = time function for period $t+\tilde{\iota}$.

The coefficients in the models are revised using equation (5)

$$\underline{\hat{a}}(t) = L\,\underline{\hat{a}}(t-1)' + h\,(x(t) - \hat{x}(t-1)) \tag{5}$$

where:

L = n x n transition matrix which adjust the coefficients for changes in the origin of time, and

h = n x 1 smoothing vector used to adjust the estimates of the coefficients for forecast errors.

For the constant model the coefficients are revised as follows:

$$\hat{a}(t) = 1\,\hat{a}(t-1) + h\,(x(t) - \hat{x}(t-1)) \tag{6}$$

and the forecast is developed using equation (7).

$$\hat{x}(t) = \hat{a}(t) \tag{7}$$

With the linear model, equation (8) is used to revise the coefficients and the forecast is developed using equation (9) where $\tilde{T}$ is the number of periods into the future for which a forecast is desired.

$$\begin{bmatrix} \hat{a}_1(t) \\ \\ \hat{a}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \hat{a}_1(t-1) \\ \\ \hat{a}_2(t-1) \end{bmatrix} + \begin{bmatrix} h_1 \\ \\ h_2 \end{bmatrix} x(t) - \hat{x}(t-1) \tag{8}$$

$$\hat{x}(t+\tilde{T}) = [\hat{a}_1(t), \hat{a}_2(t)] \begin{bmatrix} 1 \\ \tilde{T} \end{bmatrix} \tag{9}$$

In this study $\tilde{T}$ only takes on a value of 1. The reason for this is that only annual forecasts probably will be reported to the public.

The values of the smoothing vector, h, are based on a discount factor equal to .10. In choosing a value for the smoothing vector the best approach would be to simulate the historical data and select the value that minimizes the standard error of the estimate or the absolute error. Due to the large number of forecasts needed in this paper the simulation approach was not utilized. The value of .10 was selected since it has been considered a "general utility value" used most commonly.*

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*R. G. Brown, *Smoothing Forecasting and Prediction of Discrete Time Series*, Englewood Cliffs, New Jersey, Prentice Hall, Inc., 1963, p. 179. Higher values of the discount factor should be used in situations where the true coefficients can change by appreciable fractions of the noise included in the observations.